

Coupled layered superconductor as a system of 2D Coulomb particles of two kinds.

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(Dated: February 1, 2008)

It is shown that the Josephson subsystem of the Lawrence-Doniach model of layered superconductors in the London approximation can be presented as a system with variable number of classical Coulomb particles. This allows us to consider the vortex system of a coupled layered superconductor as the system of these particles and 2D-vortices interacting with each other. The grand partition function of the system was written and transformed into the form of field one. Thermodynamical properties of the model obtained was studied. It is found that there is no a phase transition in the system. Instead of this the model demonstrates the crossover from a low temperature 3D behavior to high temperature 2D one which can look as a phase transition for experimental purposes.

PACS numbers: 74.20.-z, 74.25 Bt, 74.72.-h, 64.60 Ak

I. INTRODUCTION

Behavior of the vortex system of coupled layered superconductors (SC) is one of the most interesting and difficult problem in the thermodynamics of quasi two dimensional (2D) systems. It demonstrates dimensional crossover and other issues of dimensionality which are of perpetual interest in statistical mechanics.

The Berezinskii-Kosterlitz-Thouless (BKT) phase transition^{1,2} in the 2D-vortex system was proposed to explain results of the experiment on measuring of the current-voltage characteristics of BISCO monocrystal³. Such a type of transition takes place in 2D systems in which topological defects can exist. They can be considered as 2D Coulomb particles whose interaction energy logarithmically depends on a distance between them. Layered SC's without Josephson coupling between layers are 2D systems. 2D-vortices are topological defects in these systems. 2D vortex-antivortex dipoles of a finite size can arise as thermal excitations at any temperatures and they dissociate into a gas of free vortices above BKT transition temperature $T > T_{KT}$. Properties of these systems are investigated very well⁴.

But in real layered SC's layers are coupled due to the Josephson effect. They can not be considered as 2D systems. Simplest thermal excitations in coupled layered SC's arise in the form of two 2D-vortices of opposite orientation localized in the same layer which magnetic flux is closed by two Josephson vortices or fluxons. The interaction energy of 2D-vortices in this case logarithmically depends on a distance for a small dipole size and asymptotically tends to linear dependence for a large size⁵. Investigation of such systems is much more difficult and less developed.

One of the way which was used to analyze the problem consists in studying the thermodynamic of the system of classical particles with the pair interaction potential containing both logarithmic and linear terms (see for example Refs. [6,7,8]). This approach takes into account the linear tension of Josephson vortices as a portion of a two-particle interaction energy only while to describe the problem correctly it is necessary to take into consid-

eration a Josephson vortex subsystem as an independent participant of thermodynamic processes which has own configuration energy, entropy, self-energy and interacts with the 2D vortex subsystem.

The other approach to the problem is to investigate the anisotropic 3D XY-model. It was shown that excitations of the same type as in coupled layered SC can exist in this model^{9,10,11}. The anisotropic 3D XY-model was studied numerically using Monte Carlo simulations^{10,12} and analytically^{13,14}. Results of this investigations show that the model undergoes the phase transition of the BKT type at temperature T_c higher then T_{KT} of a planar model and coupling of adjacent layers vanishes at the same temperature.

The attempt to solve the same problem, which our work is devoted to, was made in Ref. [15]. The authors base their approach on the quantum 1D sine-Gordon model and use the connection of this model with the 2D Coulomb gas system. The partition function of the model is represented in terms of a functional integral over two non-commuting variables. The model behavior is studied by means of a perturbative renormalization group (RG) approach. The RG recursion relations (RR) derived in the work have not a fixed point. However, authors analyzing RR features come to conclusion that the system undergoes phase transition which temperature T_d is controlled by 2D-vortex core energy.

The other approach to the problem discussed was proposed by Pierson and Valls¹⁶. The authors considered the model which they called a model of XY layers with Lawrence-Doniach (L-D) type interlayer couplings. It consists of 2D sine-Gordon models in each layers and additional cosine terms with difference of phases of adjacent layers, which describe the Josephson coupling. To study the model behavior authors used RG approach previously expanding the additional cosine term and keeping the quadratic summand only. The RG RR's have not a fixed point at a small value of the coupling constant but authors give arguments for existing of that at a larger value beyond the range of validity of RR's.

From our point of view such an approach to analyze the model is inadequate to the problem discussed. As

Nándori et al.¹⁷ showed in the case of the quadratic interlayer coupling the model is equivalent to the system of classical particles with long-range pair interaction. So, such an approach excludes from consideration the contribution of the entropy of the Josephson subsystem and the linear tension in the 2D-vortex interaction energy.

In this work we show that the model considered by Pierson and Valls¹⁶ follows from the L-D model¹⁸ in the London approximation and that its properties is determined by behavior of order parameter singular points of two kinds. They can be considered as classical Coulomb particles which can be associated with 2D-vortices and the Josephson subsystem.

The paper is organized as follow. In Sec. II we formulate the L-D model in the London approximation which describes the system of Josephson vortices and transform it into the form available for further generalization. In Sec. III we introduce 2D-vortices in the L-D model and show the close analogy between the 2D-vortex and the Josephson systems. In Sec. IV the model of the vortex system of a coupled layered SC is constructed as a generalization of models considered in Sec. II and III. The behavior of the model is analyzed in Sec. V and VI by means of the RG approach and in the mean field (MF) approximation. Concluding remarks are presented

in Sec. VII.

II. FLUXON SYSTEM IN THE LAWRENCE-DONIACH MODEL.

A. Model transformation.

We start from the L-D model¹⁸ in the London approximation in which fluctuations of the order parameter modulus responsible for transition in a superconducting state are neglected. In this case the order parameter is normalized so that its modulus is unit and does not depend on coordinates. Such a model describes only the system of Josephson vortices in a layered superconductor because 2D-vortices are associated with singular points in which the order parameter turns into zero. In this section we transform the model into the form suitable for its generalization and discuss briefly its thermodynamic properties.

The Hamiltonian is the functional of the phase θ of the order parameter and the vector-potential \mathbf{A} of magnetic field

$$H = \frac{\phi_0^2}{16\pi^3\Lambda(T)} \sum_n \int d\mathbf{r} dz \left[\left(\nabla\theta_n(\mathbf{r}) - \frac{2\pi}{\phi_0} \mathbf{A}(\mathbf{r}, z) \right)^2 - \frac{2}{\lambda_J^2} \cos(\Omega_n(\mathbf{r})) \right] \delta(z - ns) + \int d\mathbf{r} dz \frac{[\nabla \times \mathbf{A}(\mathbf{r}, z)]^2}{8\pi}. \quad (1)$$

Here ϕ_0 is the magnetic flux quantum, $\Lambda(T) = 2\lambda_{ab}(T)/s$, $\lambda_J = s\lambda_c/\lambda_{ab}$, $\lambda_{ab}(T)$ and $\lambda_c(T)$ is the London penetration depths parallel and perpendicular to layers, s is the period of layered structure and $\Omega_n(\mathbf{r}) = \theta_{n+1}(\mathbf{r}) - \theta_n(\mathbf{r}) - (2\pi/\phi_0) \int_{ns}^{(n+1)s} dz A_z(\mathbf{r}, z)$ is the gauge invariant phase difference. The partition function of the model can be written in terms of a functional integral as

$$Z = \int D\theta D\mathbf{A} \exp \{ -\beta H[\theta, \mathbf{A}] \}, \quad (2)$$

where $\beta = 1/T$. The vector potential \mathbf{A} in this expression is a Gaussian variable and can be integrated out. The variable \mathbf{A} is a gauge field and this must be taken into account on calculating. In the case considered it is convenient to choose the gauge condition $A_z = 0$. Resulting partition function is

$$Z = \int D\theta \exp \left\{ -\frac{1}{2J} \sum_{n,n'} \int d\mathbf{r} \frac{\nabla\theta_n(\mathbf{r}) K_{n,n'} \nabla\theta_{n'}(\mathbf{r})}{2\pi} + 2y_f \sum_n \int \frac{d\mathbf{r}}{a^2} \cos(\theta_{n+1}(\mathbf{r}) - \theta_n(\mathbf{r})) \right\} \quad (3)$$

where $J = 4\pi^2\Lambda T/\phi_0^2$, y_f is the fluxon fugacity which is proportional to Josephson critical current density and the operator

$$K_{n,n'} = \delta_{n,n'} - \int_{-\pi}^{\pi} \frac{dk}{2\pi} \frac{\cos(k(n-n'))}{1 + \frac{\Lambda}{s} k^2}$$

takes into account magnetic interaction between layers. In systems we are interested the condition $s \ll \lambda_{ab}$ is fulfilled and the magnetic interaction leads to small corrections of order s^2/λ^2 and does not change results qualitatively. Therefore for the sake of simplicity we will omit all over the paper these corrections and will take $K_{nn'} = \delta_{nn'}$. For convenience of further references we write the partition function of this reduced model because it is the initial point of our constructions

$$Z = \int D\theta \exp \left\{ -\frac{1}{2J} \sum_n \int d\mathbf{r} \frac{(\nabla\theta_n(\mathbf{r}))^2}{2\pi} + 2y_f \sum_n \int \frac{d\mathbf{r}}{a^2} \cos(\theta_{n+1}(\mathbf{r}) - \theta_n(\mathbf{r})) \right\} \quad (4)$$

The field partition function (4) can be transformed into the form of that of a gas of classical Coulomb particles.

In order to perform such a mapping we follow the scenario used by Nandori et al.¹⁷. With this aim in view we carry out the change of the variable $\theta = \sqrt{J}\varphi$ and

present the exponential function with the cosine term in its argument as a product of two exponents

$$\exp \left\{ 2y_f \sum_n \int \frac{d\mathbf{r}}{a^2} \cos \left[\sqrt{J}(\varphi_{n+1} - \varphi_n) \right] \right\} = \prod_n \exp \left\{ y_f \int \frac{d\mathbf{r}}{a^2} e^{i\sqrt{J}(\varphi_{n+1} - \varphi_n)} \right\} \exp \left\{ y_f \int \frac{d\mathbf{r}}{a^2} e^{-i\sqrt{J}(\varphi_{n+1} - \varphi_n)} \right\}.$$

Then we expand them into Taylor series. In the result after obvious change of summation we obtain the expression required

$$Z = \prod_n \left[\sum_{M_{n\pm}} \frac{1}{M_{n+}!} \frac{1}{M_{n-}!} \left(e^{\beta\mu_f} \int \frac{d\mathbf{r}_{j_n}}{\xi^2} \right)^{M_{n+}+M_{n-}} \right] \times \int D\varphi \exp \left\{ -\frac{1}{2} \sum_n \int d\mathbf{r} \frac{(\nabla\varphi_n(\mathbf{r}))^2}{2\pi} + i \sum_n \sum_{j_n=1}^{M_{n+}+M_{n-}} q_{j_n} (\varphi_{n+1}(\mathbf{r}_{j_n}) - \varphi_n(\mathbf{r}_{j_n})) \right\}. \quad (5)$$

In this expression $q_{j_n} = \pm\sqrt{J}$ are charges of the fluxon particles, $M_{n+}(M_{n-})$ are numbers of particles with a positive (negative) charge in n th layer, μ_f is the chemical potential of the particles and length a is replaced by the correlation length ξ .

The particles under consideration, apparently, are not real ones. They look as dipoles which poles have the same in-plane positions but are located in adjacent layers and can be presented as a point current source and sink. In fact, such a particle is a piece of a current line connecting adjacent layers.

Further transformation of the partition function is possible if we notice that φ is the Gaussian variable and can be integrated out. Such a procedure leads to the partition function of the system of classical particles with a pair interaction

$$Z = \prod_n \left[\sum_{M_{n\pm}} \frac{1}{M_{n+}!} \frac{1}{M_{n-}!} \left(e^{\beta\mu_f} \int \frac{d\mathbf{r}_{j_n}}{\xi^2} \right)^{M_{n+}+M_{n-}} \right] \times \exp \left\{ -\frac{1}{2} \sum_{n,n'} \sum_{j_n,j_{n'}} q_{j_n} q_{j_{n'}} v_{nn'}(|\mathbf{r}_{j_n} - \mathbf{r}_{j_{n'}}|) \right\}. \quad (6)$$

The potential $v_{nn'}$ corresponds to 2D Coulomb interaction of above-mentioned fluxon particles

$$v_{nn'}(\mathbf{r}_{j_n} - \mathbf{r}_{j_{n'}}) = \ln \frac{|\mathbf{r}_{j_n} - \mathbf{r}_{j_{n'}}|}{\xi} \left(\delta_{n,n'} - \frac{1}{2}\delta_{n,n'-1} - \frac{1}{2}\delta_{n,n'+1} \right). \quad (7)$$

There is another way to obtain the partition function (6). We can start from Eq. (4) with $y_f = 0$ and assume that the order parameter possesses singular points

in which the conditions

$$\begin{aligned} (\nabla, \nabla\theta_{n'}(\mathbf{r} - \mathbf{r}_{j_n})) &= \pm 2\pi J \delta(\mathbf{r} - \mathbf{r}_{j_n}) (\delta_{n+1,n'} - \delta_{n,n'}), \\ [\nabla, \nabla\theta_n(\mathbf{r})] &= 0 \end{aligned} \quad (8)$$

are satisfied. It is easy to show that the energy of interaction of singular points defined by these conditions is the same Coulomb potential (7) as that of particles discussed.

Thus, the field model (4) of fluxon subsystems can be presented as the system with variable number of classical particles with pair interaction. Such a point of view on the fluxon system will be very useful in Sec.IV to construct the model in which fluxon and 2D-vortex subsystems are joined.

B. Thermodynamic properties of the model.

Peculiarities of behavior of the model will reveal themselves in the properties of the joined model. To better understand them we discuss briefly the thermodynamic properties of the fluxon subsystem. The main interest will be directed to an order and temperature of the phase transition. Similar model was studied by Horovitz¹⁹. He showed that the second order phase transition of the BKT type takes place in the system. Here we discuss results of our examination of the model which are very close to Horovitz ones.

We perform the momentum space RG study of Eq.(4). RR's are a set of two equations in parameters of the initial Hamiltonian

$$\frac{dy_f}{d\tau} = (2 - J)y_f, \quad (9)$$

$$\frac{dJ}{d\tau} = -8\pi^2 J^3 y_f^2. \quad (10)$$

The phase portrait of the set is plotted in Fig. 1(a). It is the conventional picture of flows in the vicinity of a fixed point of the saddle type which is defined by the conditions ($J = 2$, $y_f = 0$). Existence of the fixed point means that a second order phase transition takes place in the system at the temperature defined by the expression

$$T_f = \frac{\phi_0^2}{2\pi^2\Lambda(T_f)} \quad (11)$$

which follows from the condition $J = 2$. This temperature is placed above than that of the BKT transition T_{KT} in the 2D-vortex system of decoupled layered SC and is much more close to the temperature T_c of the transition of a sample into a superconducting state. Another feature of the transition is the directions of the RG flows. They are shown by arrows in Fig. 1(a) and, as it will be shown in next section (see Fig. 2(a)), are opposite to those of 2D-vortex system.

The fugacity y_f turns to zero in the fixed point. As y_f is proportional to Josephson critical current density this means that it vanishes in the transition point $T = T_f$ too and superconducting layers becomes decoupled at high temperatures $T > T_f$.

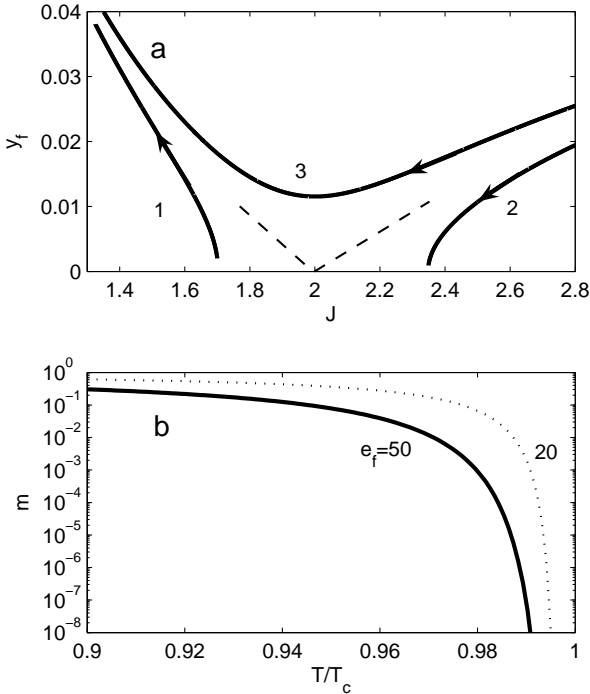


FIG. 1: (a) Phase trajectories of the fluxon system RG equations in the vicinity of the fixed point ($J = 2$, $y_f = 0$) of saddle type. (b) Temperature dependence of the fluxon particle concentration for the systems with different anisotropy $e_f = 50$ and 20 not far from the phase transition

Let us consider also the temperature dependence of the concentration of fluxon particles. We found it in

the MF approximation. The free energy of the system per the unit area of one layer was obtained from Eq.(6) by means of the thermodynamic perturbation theory in the ring approximation²⁰. Under the suppositions that the system is neutral, equilibrium numbers of particles doesn't depend on the layer index ($M_{n+} = M_{n-} = M$) and the screening length $\delta_f \gg \xi$ the expression for the dimensionless free energy takes the form

$$f = \frac{\beta F \pi \xi^2}{S} = 2m(\ln(m) - 1) - Jm \ln(4Jm) + mJ e_f. \quad (12)$$

Here $m = \pi \xi^2 M / S$ is the dimensionless particle concentration, S is the layer area, and $1/\sqrt{4Jm} = \delta_f/\xi$ is the dimensionless Debye screening length. The last term is, in fact, the chemical potential one. It controls the system anisotropy and we consider e_f as a phenomenological parameter. The free energy (12) reaches its minimal value at the particle concentration

$$m = \left(\frac{4J}{e^{e_f} - 1} \right)^{\frac{J}{2-J}}. \quad (13)$$

This solution is stable at $J < 2$ ($T < T_f$). In the point $J = 2$ it loses the stability and at $J > 2$ ($T > T_f$) equilibrium value of the concentration is $m = 0$. So, the MF approximation leads to the same temperature (11) of the phase transition in correspondence with RG results.

The temperature dependence of the equilibrium particle concentration is plotted in Fig. 1(b) for systems with different anisotropy. To derive this picture the model temperature dependence of the London length $\lambda_{ab}(T) \sim (1 - T/T_c)^{-1/2}$ was used and parameters of the model were chosen in such a way to fix $T_{KT}/T_c = 0.98$. These options result in $T_f/T_c \approx 0.998$. It is seen that the free particle concentration m in a more anisotropic system ($e_f = 50$) always is less then that in a less anisotropic ($e_f = 20$). We can see also some correlations between temperature dependencies of the free fluxon particle concentration and critical density of the Josephson current: both of them reach their maximum values at $T = 0$, both monotonically decrease when the temperature increases, both are greater in less anisotropic systems, both turn into zero at $T = T_f$ and vanish at $T > T_f$.

III. 2D-VORTEX SYSTEM IN THE LAWRENCE-DONIACH MODEL.

A. Model constructing and transformation

To describe the 2D-vortex system in the L-D model we use the Eq.(4) with $y_f = 0$. 2D-vortices are topological excitations and can be introduced in the model as singular points of the order parameter placed in positions \mathbf{r}_{j_n} of the n th layer and defined by the conditions

$$[\nabla, \nabla \theta_n(\mathbf{r})] = 2\pi s_{j_n} \mathbf{z} \delta(\mathbf{r} - \mathbf{r}_{j_n}), \quad (\nabla, \nabla \theta_n(\mathbf{r})) = 0. \quad (14)$$

The symbol $s_{j_n} = \pm 1$ means the sign of a charge of the vortex placed in the position \mathbf{r}_{j_n} of the n th plane and \mathbf{z} is the unit vector in z direction. The energy of interaction of two vortices defined in such a way and placed in the points \mathbf{r}_{i_n} and \mathbf{r}_{j_n} of the same n th layer is

$$\begin{aligned} U(\mathbf{r}_{i_n} - \mathbf{r}_{j_n}) &= -Q_{i_n} Q_{j_n} u(\mathbf{r}_{i_n} - \mathbf{r}_{j_n}) \\ &= -Q_{i_n} Q_{j_n} \ln \left(\frac{|\mathbf{r}_{i_n} - \mathbf{r}_{j_n}|}{\xi} \right), \end{aligned} \quad (15)$$

where $Q_{j_n} = s_{j_n}/\sqrt{J}$ is the 2D-vortex charge. The interaction potential (15) is the 2D Coulomb one and obeys

the 2D Poisson equation

$$-\Delta u(\mathbf{r}) = 2\pi\delta(\mathbf{r}). \quad (16)$$

One can write the partition function of the system of 2D-vortices considering them as classical massless particles with the interaction potential (15)

$$Z = \prod_n \left[\sum_{N_{n+}} \frac{1}{N_{n+}!} \frac{1}{N_{n-}!} \left(e^{\beta\mu_v} \int \frac{d\mathbf{r}_{j_n}}{\xi^2} \right)^{N_{n+}+N_{n-}} \right] \exp \left\{ -\frac{1}{2} \sum_n \sum_{i_n j_n} Q_{i_n} Q_{j_n} u(\mathbf{r}_{i_n} - \mathbf{r}_{j_n}) \right\}. \quad (17)$$

Here $N_{n\pm}$ are the numbers of particles with the positive and negative sign of charge and μ_v is the 2D-vortex chemical potential. This partition function can be transformed into the forms of that of a system of particles into self-consistent field and that of a field system²¹. With

this aim in view it is need to multiply the exponential function in Eq.(17) by the unit, which is represented as the ratio of two identical Gaussian integrals over a field variable φ

$$\exp \left\{ -\frac{1}{2} \sum_n \sum_{i_n j_n} Q_{i_n} Q_{j_n} u_n(\mathbf{r}_{i_n} - \mathbf{r}_{j_n}) \right\} \frac{1}{Z_0} \int D\varphi \exp \left\{ -\frac{1}{2} \sum_n \int d\mathbf{r} \frac{(\nabla\varphi_n)^2}{2\pi} \right\}. \quad (18)$$

The integral in the denominator is just a normalizing constant Z_0 . The next step is to change of the variable in the numerator

$$\varphi_n(\mathbf{r}) = \varphi_n(\mathbf{r}) + i \sum_{j_n} Q_{j_n} u_n(\mathbf{r} - \mathbf{r}_{j_n})$$

which is chosen in such a way that after integrating by

parts over \mathbf{r} and using Eq.(16) it leads to compensation of the sum of interaction potentials in the power of exponential function (17). As a result, the partition function (17) can be rewritten in the form of that of a system of particles in self-consistent field

$$Z = \prod_n \left[\sum_{N_{n+}} \frac{1}{N_{n+}!} \frac{1}{N_{n-}!} \left(e^{\beta\mu_v} \int \frac{d\mathbf{r}_{j_n}}{\xi^2} \right)^{N_{n+}+N_{n-}} \right] \int D\varphi \exp \left\{ -\frac{1}{2} \sum_n \int d\mathbf{r} \frac{(\nabla\varphi_n)^2}{2\pi} + i \sum_n \sum_{j_n} Q_{j_n} \varphi_n(\mathbf{r}_{j_n}) \right\} \quad (19)$$

In the Eq.(19) we can take the summation over particle numbers N_{n+} and N_{n-} to obtain the field sine-Gordon model. The substitution of the variable in the functional integration $\varphi = \theta/\sqrt{J}$ returns us to the initial variable θ , which is the phase of the order parameter. The partition function finally takes the form

$$Z = \int D\theta \exp \left\{ -\frac{1}{2J} \sum_n \int d\mathbf{r} \frac{(\nabla\theta_n)^2}{2\pi} + 2y_v \sum_n \int \frac{d\mathbf{r}}{a^2} \cos \left(\frac{1}{J} \theta_n(\mathbf{r}) \right) \right\}.$$

Here $y_v = \exp\{\beta\mu_v\}$ is the 2D-vortex fugacity and nonessential constant Z_0 is omitted.

Thus, the 2D-vortex system of a layered superconductor can be represented in the same three forms as the fluxon one. Below we discuss briefly thermodynamical properties of the 2D-vortex system to compare them with those of the fluxon one.

B. Thermodynamic properties of the model.

Thermodynamical properties of this model was studied very much and are well known^{4,19}.

The momentum space RG approach based on Eq. (20) gives the RR's

$$\frac{dy_v}{d\tau} = \left(2 - \frac{1}{2J}\right) y_v, \quad (20)$$

$$\frac{dJ}{d\tau} = \frac{4\pi^2}{J} y_v^2. \quad (21)$$

Existence of the fixed point ($J = 1/4$, $y_v = 0$) of the saddle type means that the second order phase transition takes place in the system at the temperature

$$T_{KT} = \frac{\phi_0^2}{16\pi^2\Lambda(T_{KT})}. \quad (22)$$

This is the conventional BKT transition. It is easy to see comparing Eqs. (22) and (11) that $T_{KT} < T_f$. The phase portrait of the Eqs. (20) and (21) is plotted in Fig. 2(a). Comparison of Figs. 1(a) and 2(a) shows that RG flows of the fluxon and the 2D-vortex systems are oppositely directed.

To find temperature dependence of the 2D-vortex concentration we used the MF approximation. The free energy of the system as a function of the vortex number can be obtained from the partition function Eq. (17) by means of the thermodynamic perturbation theory in the ring approximation²⁰. Under supposition that the system is neutral the dimensionless free energy per the area unit $\pi\xi^2$ of one layer takes the form

$$f = 2n(\ln n - 1) - \frac{n}{2J} \left(\ln \frac{4n}{J} - 1 \right) + n \frac{e_v}{2J}. \quad (23)$$

Here $n = \pi\xi^2 N/S$ is the dimensionless vortex concentration, $\sqrt{J/4n} = \delta_v/\xi$ is the dimensionless Debye screening length, $e_v/4J$ is the vortex core energy.

The equilibrium vortex concentration obeys the equation

$$n = \left(\frac{4}{J} e^{-e_v} \right)^{\frac{1}{4J-1}}. \quad (24)$$

This solution is stable at $T > T_{KT}$. At $T = T_{KT}$ it becomes zero and loses the stability at $T < T_{KT}$ where zero solution becomes stable. Temperature dependence of the concentration is shown in Fig. 2(b). To obtain this

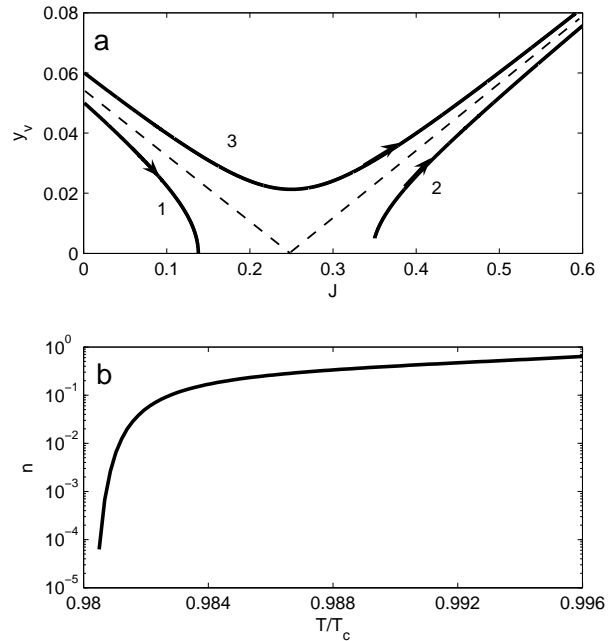


FIG. 2: (a) Phase trajectories of the 2D vortex system RG equations in vicinity of the fixed point ($J = 1/4$, $y_v = 0$). (b) Temperature dependence of the free 2D-vortex concentration.

picture we used model temperature dependence $\lambda_{ab}(T) \sim (1 - T/T_c)^{-1/2}$ and chose model parameters to specify $T_{KT}/T_c = 0.98$.

It is interesting to compare temperature dependencies of fluxon particles Fig. 1(b) and 2D-vortices Fig. 2(b) concentrations. A state with a finite concentration of 2D-vortices is stable in the temperature range $T_{KT} < T < T_c$ which is located higher the phase transition temperature while that of fluxon particles is stable at lower temperatures $0 < T < T_f$.

So, we considered two basic models. We found their common features and revealed differences between them. By this we finish consideration of the previous results. We have all we need to construct the model of the vortex system of a coupled layered superconductor. This is the subject of next section.

IV. MODEL OF INTERACTING 2D-VORTEX AND FLUXON SYSTEM.

Now as a result of previous consideration it is easy to see close analogy of the fluxon and 2D-vortex subsystems of a layered SC. Each of them can be presented either as a field model of a sine-Gordon type or as a system of classical particles with a pair Coulomb interaction. Difference between them consists in the structure and the value of "charge" of the particles only. But our aim is to develop the model describing a coupled layered SC

which contains both these subsystems. Therefore we will consider a coupled layered SC as a system consisting of two kinds of 2D Coulomb particles, namely, fluxon particles and 2D-vortices, interacting with each other. Such a view on layered SC's allow us to obtain the partition function of their vortex system to study thermodynamic properties.

There are two ways to solve this problem. One can act in the complete analogy with constructing of the 2D-vortex subsystem in Sec. III taking into account singular points of the order parameter defined by both conditions

(8) and (14) to obtain 2D-vortex, fluxon particle and 2D-vortex - fluxon particle pair interaction potentials and write a partition function.

The other way is to construct the partition function of the system of particles of both kinds in self-consistent field. Really, existence of the interaction between the particles means that all of them produce the same field and interact with it in accordance with their structures and charges. In the analogy with Eqs. (5) and (19) we can write

$$Z = \prod_n \left[\sum_{N_{n\pm}} \frac{1}{N_{n+}!} \frac{1}{N_{n-}!} \left(e^{\beta\mu_v} \int \frac{d\mathbf{r}_{j_n}}{\xi^2} \right)^{N_{n+}+N_{n-}} \sum_{M_{n\pm}} \frac{1}{M_{n+}!} \frac{1}{M_{n-}!} \left(e^{\beta\mu_f} \int \frac{d\mathbf{r}_{\alpha_n}}{\xi^2} \right)^{M_{n+}+M_{n-}} \right] \times \int D\varphi \exp \left\{ -\frac{1}{2} \sum_n \int d\mathbf{r} \frac{(\nabla\varphi_n)^2}{2\pi} + i \sum_n \sum_{j_n} Q_{j_n} \varphi_n(\mathbf{r}_{j_n}) + i \sum_n \sum_{\alpha_n} q_{\alpha_n} (\varphi_{n+1}(\mathbf{r}_{\alpha_n}) - \varphi_n(\mathbf{r}_{\alpha_n})) \right\}. \quad (25)$$

Next step is to carry out the sums over all numbers of particles. These sums are easy to calculate because the result of integration over particle coordinates \mathbf{r}_{j_n} and \mathbf{r}_{α_n} do not depend on particle indices and sums are just

Taylor series of exponential functions. As a result, the partition function after change of a variable $\varphi = \theta/\sqrt{J}$ takes the form

$$Z = \int D\theta \exp \left\{ -\frac{1}{2J} \sum_n \int d\mathbf{r} \frac{(\nabla\theta_n(\mathbf{r}))^2}{2\pi} + 2y_v \sum_n \int \frac{d\mathbf{r}}{a^2} \cos\left(\frac{1}{J}\theta_n(\mathbf{r})\right) + 2y_f \sum_n \int \frac{d\mathbf{r}}{a^2} \cos(\theta_{n+1}(\mathbf{r}) - \theta_n(\mathbf{r})) \right\}. \quad (26)$$

The expression obtained is the model we developed. It describes the fluxon subsystem, the 2D-vortex subsystem as well as interaction between them and is, in fact, the reduced L-D model in which order parameter fluctuations is neglected. This model includes the initial models, which was the base to develop the model, as limiting cases. In the case $y_v = 0$ the model (26) reverts back to that of the fluxon system (4) and the condition $y_f = 0$ convert it into Eq. (20) which is the partition function of the 2D-vortex subsystem.

The model considered coincides actually with that proposed by Pierson et al.¹⁶. To investigate the model authors expanded the second cosine term and performed a RG analysis of the reduced model. Nándori et al.¹⁷ showed that the reduced model is equivalent to a gas of topological defects with long-range interaction potentials. But as it was shown in this section the model 26 is equivalent to the gas of classical Coulomb particles of two kinds. In contrast to work¹⁶ we study the model taking this into account.

V. RENORMALIZATION GROPE ANALYSIS.

We derived the perturbative RG equations in the parameters of the Hamiltonian of the model (26) by means of the momentum space approach. Recursion relations we obtained

$$\frac{dJ}{d\tau} = \frac{4\pi^2}{J} y_v^2 - 8\pi^2 J^3 y_f^2, \quad (27)$$

$$\frac{dy_v}{d\tau} = \left(2 - \frac{1}{2J} \right) y_v, \quad (28)$$

$$\frac{dy_f}{d\tau} = (2 - J) y_f \quad (29)$$

takes into account renormalization of the parameters of the initial Hamiltonian only. It is easy to see that the set of equations inherits the structures of sets (9-10) and (20-21) as well as the model (26) inherits the structures of models (4) and (20). Equations in fugacities y_v and y_f are just identical and the right-hand side (r.h.s.) of

Eq. (27) is the sum of those of Eqs. (9) and (20). Such a structure of the equations results in that some details of the model behavior are similar to that of the fluxon or 2D-vortex models but another ones are quite different.

Behavior of the set considered is much more complicated. Main peculiar features of the set are that the number of independent dynamical variables rises from 2 to 3 and the r.h.s. of Eq. (27) is not sign-definite. In the result the recursion relations (27)-(29), contrary to that of the fluxon system (9)-(10) and the 2D-vortex one (20)-(21), have not a fixed point and thus the system does not undergo a second order phase transition.

Typical trajectories of the RG recursion relations are shown in Fig. 3 as projections on planes $y_v - J$ and $y_f - J$. The initial conditions are chosen in such a way to get the curves which projections behavior, in a large extent, are similar to those of corresponding trajectories of the fluxon and the 2D-vortex models. It is easy to see that the projection of trajectory marked by 1 in Fig. 3(a) at small values of the scale variable τ behaves analogous to the trajectory 1 in Fig. 2(a). But at some finite value of τ second term in r.h.s. of Eq. (27) becomes dominated and the derivative of J changes the sign from plus to minus. It is a turning point of the trajectory. After this point the analogy mentioned disappears but instead analogy between the projection 1 in Fig. 3(b) and the trajectory 1 in Fig. 1(a) appears. Similar analogies can be find between the projections of trajectory 2 in Fig. 3 and the trajectories 2 in Figs. 1(a) and 2(a). There are else two analogies between phase portraits of these systems. The projection 3 in Fig. 3(a) is similar to the trajectory 3 in the phase portrait of the RG equations of the 2D-vortex system, and the projection 4 in Fig. 3(b) behaves analogously to the trajectory 3 in the phase portrait of the RG equations of the fluxon system.

Such an analogy between phase portraits of the model considered and the models of independent 2D-vortex and fluxon systems allows us to believe that there are some similarity between behavior of these systems. But this analogy is observed in the space of parameters of Hamiltonian and it does not allow us to draw a valid conclusion about behavior of the vortex system in the space of thermodynamical variables. Let us consider the phase portrait shown in Fig. 3 from another point of view.

The RG flow can be characterized, in addition to forms of trajectories and direction of the system motion along them, also by a rate of the system motion. If the recursion relations have a fixed point the rate of motion along the trajectory is minimal in the position which is nearest to this point and asymptotically tends to zero when a trajectory approaches to it. Such a situation takes place in the RG flows of the fluxon Fig. 1(a) and the 2D-vortex Fig. 2(a) systems.

In the case of the model proposed the situation is different. The RG flow Fig. 3 does not have a fixed point. Instead, it has a slow point in which a rate of motion along a trajectory is lowest. This point can mimic a fixed point for experimental purposes. The properties of

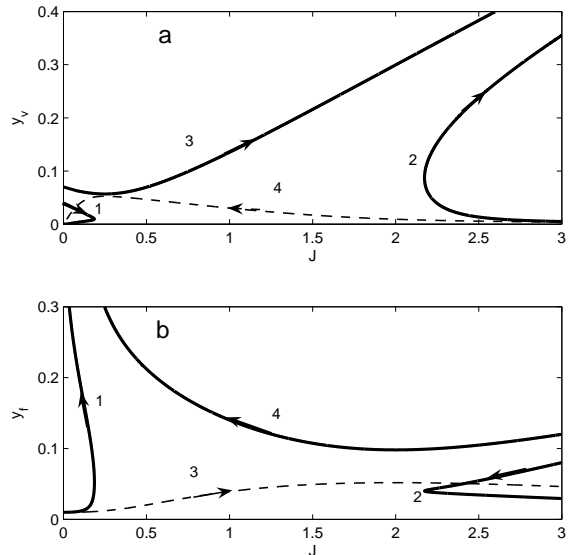


FIG. 3: Projections of phase trajectories of the model RG equations.

models which demonstrate similar behavior were investigated by Zumbach²². He termed such a situation as an "almost second order phase transition".

From such a point of view we can suppose that the model asymptotically behaves almost the free fluxon and the 2D-vortex systems in regions of low ($T < T_{KT}$) and high ($T > T_f$) temperature and demonstrates the crossover from low temperature 3D behavior to high temperature 2D one in vicinity of temperature T_s corresponding to a slow point. The turning points of trajectories of different kinds (1-4 in Fig.3) draw together in close vicinity of a slow point. This allow us to evaluate the position of this point. Direct numerical calculation of trajectories passing in the vicinity of slow point shows that such a point corresponds to $J \approx 0.7$. The crossover temperature which can be obtained from this relation obeys the equation

$$T_s = 0.7 \frac{\phi_0^2}{4\pi^2 \Lambda(T_s)}. \quad (30)$$

But a perturbative RG is not a very good approach to find a slow point because the position of this point can be dependent on a cut-off procedure. So, the expression (30) can be considered as a rough evaluation of the crossover temperature only. The fact in which we can be sure is that $T_{KT} < T_s < T_f$ or the same $1/4 < J_s < 2$.

In completion of the section we note that a RG approach is the examination of the hypothesis about a scaling invariance of the model. In the case considered the RG does not have a fixed point and, thus the hypothesis is not prove to be true, in contrast to the independent 2D-vortex and fluxon models. In this situation we have to use a different approach to understand reasons of the

scaling invariance breaking and to investigate the model behavior in more details. This is a subject of the next section.

VI. MEAN FIELD ANALYSIS.

In this section we will consider the model as a two subsystems of classical particles with the long-range

Coulomb interactions (7) and (15) interacting with each other. To obtain the free energy of the model in a MF approximation we use the method developed in Ref. [23]. This method corresponds to the ring approximation in the case of the Coulomb gas²⁰. The system free energy obtained is a function of concentrations of both 2D-vortices (n) and fluxon particles (m). Under the assumption that the system is neutral ($N_{n+}(M_{n+}) = N_{n-}(M_{n-}) = N(M)$) the free energy takes the form

$$f = 2n(\ln n - 1) + 2m(\ln m - 1) - \left(\frac{n}{J} + 2mJ\right) \ln \left(\sqrt{\frac{n}{J}} + \sqrt{\frac{n}{J} + 4mJ}\right) + \frac{1}{2}\sqrt{\frac{n}{J}}\sqrt{\frac{n}{J} + 4mJ} + n\frac{e_v}{2J} + mJe_f. \quad (31)$$

The condition of a minimum of the free energy is two equations in equilibrium concentrations

$$\begin{aligned} 2\ln n - \frac{1}{J} \ln \left(\sqrt{\frac{n}{J}} + \sqrt{\frac{n}{J} + 4mJ}\right) + \frac{e_v}{2J} &= 0, \\ 2\ln m - 2J \ln \left(\sqrt{\frac{n}{J}} + \sqrt{\frac{n}{J} + 4mJ}\right) + J \frac{\sqrt{\frac{n}{J}} - \sqrt{\frac{n}{J} + 4mJ}}{\sqrt{\frac{n}{J}} + \sqrt{\frac{n}{J} + 4mJ}} + Je_f &= 0, \end{aligned} \quad (32)$$

The solutions of the equations obtained numerically are shown in Fig.4 as temperature dependence of the equilibrium concentrations of the 2D-vortices and fluxon particles by solid lines for more anisotropic system ($e_f = 50$) and by dotted lines for less anisotropic ($e_f = 20$). These concentrations always are above the corresponding values of the independent subsystems, which are plotted by the dashed lines, and in contrast to those are finite at any temperatures in the whole region $0 < T < T_c$.

This result is easy to understand. There is BKT phase transition in the independent systems of 2D-vortices and fluxon particles. The mechanism realizing the transition is follow. A single 2D-vortex (fluxon particle) in an infinite sample has infinite energy. As a result of this such a particle can not appear in the system as a thermal fluctuation at low temperature $T < T_{KT}$ (high one $T > T_f$). But in a many body system the competition between the configuration energy and entropy terms in the free energy (see Eqs. (12) and (23)) at $T > T_{KT}$ ($T < T_f$) leads to instability of a zero concentration state and to appearance of a finite concentration of free 2D-vortices (fluxon particles). These particles differ from single ones mentioned above because they have a finite energy due to Debye screening. Such free vortices (fluxon particles) are, in fact, quasi-particles and can appear in the system as thermal fluctuations.

The model discussed describes systems of 2D-vortices and fluxon particles interacting with each other. In the case of Coulomb particles the interaction leads to their mutual screening. Thus, a finite concentration of 2D-vortices (fluxon particles) at low temperature $T < T_{KT}$ (high one $T > T_f$) exists due to screening of them by fluxon particles (2D-vortices). This supposition is easy to

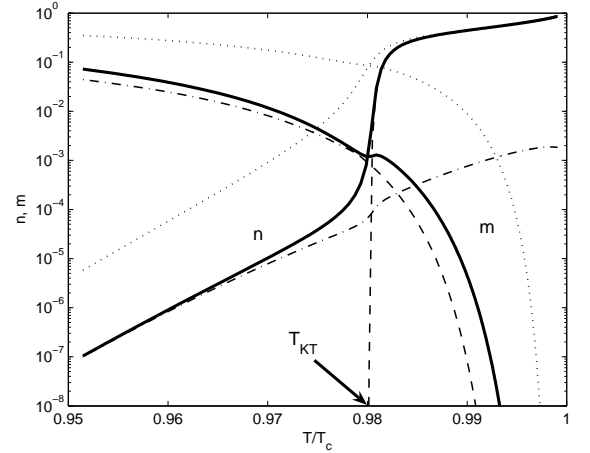


FIG. 4: Solutions of equilibrium equations are shown by solid lines for the high anisotropic system $e_f = 50$ and by dotted lines for the low anisotropic one $e_f = 20$. Dash and dash-dot lines show asymptotical solutions Eqs. (33) and (34)

verify. The equilibrium equations (32) can be simplified in the vicinity of $T = 0$ ($T = T_c$) due to condition $n \ll m$ ($m \ll n$). Asymptotical solutions of the equation take the form

$$\begin{aligned} m &= \left(\frac{4J}{e^{e_f}-1}\right)^{\frac{J}{2-J}}, \\ n &= \exp\left\{\frac{1}{4J}(\ln 4Jm - e_v)\right\} \end{aligned} \quad (33)$$

in the vicinity of $T = 0$ and

$$\begin{aligned} n &= \left(\frac{4}{J} e^{-e_v} \right)^{\frac{1}{4J-1}}, \\ m &= \exp \left\{ \frac{J}{2} \left(\ln \frac{4n}{J} - e_f \right) \right\} \end{aligned} \quad (34)$$

close to $T = T_c$.

The solutions obtained make a clear sense. The concentration m in (33) coincides with that in the independent fluxon system (13). This is sequent of a very small concentration n of 2D-vortices at low temperatures $T \ll T_{KT}$, which can be neglected in comparison with m . The concentration of 2D-vortices n in the limit considered is just the Boltzmann expression $n = \exp\{-E_v/T\}$, where E_v is the vortex energy containing two terms. Second term is the vortex core energy $e_v/4J$. First one is the energy of a 2D-vortex which diverges logarithmically in the case of a single vortex but is limited by the screening length δ_f due to a finite concentration m of fluxon particles, $4mJ = \xi^2/\delta_f^2$. The Eq. (34) has an analogous structure: the 2D-vortex concentration n coincides with that in the independent system Eq. (24) and the concentration m is defined by the self-energy of a fluxon particle screened by a finite concentration of 2D-vortices, $4n/J = \xi^2/\delta_v^2$.

Asymptotical temperature dependencies of the concentrations of the particles of both kinds are in a good agreement with the numerical solution. The concentrations of 2D-vortices and fluxon particles of independent systems are plotted in Fig. 4 by dashed lines. Asymptotical dependencies of concentrations of 2D-vortices at low temperatures and fluxon particles at high temperatures are shown by dash and dot lines.

It is seen that asymptotical behavior of the model at $T \rightarrow 0$ and $T \rightarrow T_c$ is very close to that of the independent 2D-vortex and fluxon systems. The concentrations n at $T \ll T_{KT}$ and m at $T > T_f$ are exponentially small but finite. This means that there is no a phase transition in the model discussed but there is a crossover from 3D to 2D type of behavior which takes place in the temperature interval $T_{KT} < T < T_f$.

Thus results obtained in the framework of the MF approximation agree quantitatively with that of the RG analysis. In terms of the MF approximation they are conditioned by mutual screening of particles of two kinds.

The RG considers this as breaking of the scaling invariance of the model. Now we can see that the reason which leads to this is existence of two competitive lengths, namely, the screening lengths δ_v and δ_f .

VII. CONCLUSION

We constructed and studied the model of the vortex system of coupled layered superconductors which is based on the L-D one. The main idea is to consider the Josephson and 2D-vortex subsystems as the systems of singular points defined by the conditions (8) and (14). Such an approach becomes possible because both singularities interacts according to the 2D Coulomb law. So, they can be interpreted as classical massless Coulomb particles which are characterized by their charges and have different structures. The model partition function can be represented either as a grand partition function of the system with a variable number of particles of two kinds (25) or as a that of the field model (26).

The model was examined by means of the perturbative RG approach and the MF approximation. Results obtained by both methods agree qualitatively with each other. Both approaches show that there is no phase transition in the model in the whole temperature interval $0 < T < T_c$ where the model is defined. But the model behaves asymptotically at $T \rightarrow 0$ and $T \rightarrow T_c$ as independent 2D-vortex and Josephson systems. Crossover from low-temperature 3D behavior to high-temperature 2D one takes place in the interval $T_{KT} < T < T_f$.

The effect of anisotropy can be cleared by means of comparing of temperature dependencies of free particles concentrations of systems with different value of the parameter e_f in Fig. 4. One can see that more anisotropic system ($e_f = 50$) behaves closer to that of independent 2D-vortex and fluxon particle systems than less anisotropic ($e_f = 20$). If the layered system is anisotropic enough it can mimic the BKT phase transition for experimental purposes.

Acknowledgments

The author is thankful to A. E. Filippov and L. Benfatto for useful personal discussions.

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